8. V. P. Agafonov, V. K. Vertushkin, A. A. Gladkov, and O. Yu. Polyanskii, Nonequilibrium Physicochemical Processes in Aerodynamics [in Russian], Mashinostroenie, Moscow (1972).
9. E. E. Nikitin, The Theory of Elementary Atomic-Molecular Processes in Gases [in Russian], Khimiya, Moscow (1970).
10. J. G. Parker, "Vibrational relation times of O_2 with O_2 and N_2 with N_2," J. Chem. Phys., 41, No. 5 (1964).
11. J. H. Hiefer and R. W. Lutz, "Temperature dependence of vibrational relaxation times of O_2 with O," in: 15th Symposium on Combustion, Berkeley, 1966, Pittsburgh (1967).
12. E. E. Nikitin and S. Ya. Umanski, "Vibronic-to-translational energy transfer in molecular collisions," Faraday Disc. Chem. Soc., 53, No. 7 (1972).
13. H. K. Cheng, "The blunt-body problem in hypersonic flow at low Reynolds number," Paper No. 63-92, IAS, New York (1963).
14. É. A. Gershbein, S. V. Peigin, and G. A. Tirskii, "Supersonic streamlining of bodies with low and moderate Reynolds numbers," in: Itogi Nauki Tekh., Ser. Mekh. Zhidk.Gaza, Vol. 19, VINITI, Moscow (1985).
15. C. M. Ablow, S. Schechter, and W. H. Zwisler, "Node selection for two-point boundary-value problems," Trans. ASME J. Fluids. Engng., 107, No. 3 (1985).
16. W. Gibson, "The effect of ambient dissociation and species diffusion on nonequilibrium shock layer," Paper No. 63-70, IAS, New York (1963).
17. V. N. Gusev, V. P. Provotorov, and V. V. Ryabov, "The role of the physicochemical processes in problems of simulating hypersonic flows of a rarefied gas," Uchen. Zap TsAGI, 12, No. 4 (1981).
18. V. N. Gusev, M. N. Kogan, and V. A. Perepukhov, "Similarity and changes in the aerodynamic characteristics in the transition region for hypersonic flow velocities," Uchen. Zap TsAGI, 1, No. 1 (1970).
19. A. K. Artomonov, V. N. Arkhipov, and V. G. Farafonov, "Similarity criteria in the aerodynamics of a rarefied gas," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1981).

A SELF-SIMILAR PROBLEM DEALING WITH THE ONE-DIMENSIONAL COLLISION OF TWO HALF SPACES OF A NONLINEAR-ELASTIC MATERIAL

I. E. Agapov, A. M. Belogortsev, A. A. Burenin, and A. V. Rezunov

UDC 539.3

The problems associated with investigating the processes of high-speed collisions in deformable bodies is of great theoretical and applied significance. However, the solution of these problems is associated with considerable difficulties, ascribed to the wave nature of strain propagation, as well as by the need to make use of nonlinear mechanical models. Self-similar problems from nonlinear elasticity theory provide specific information regarding the behavior of bodies under conditions of intensive dynamic load, and the solution for these problems can be found analytically or comparatively easily with numerical methods. The qualitative features of the process of propagating perturbations has been studied [1], and solutions have been obtained for a series of specific problems [2-4].

In the present study we examine a one-dimensional problem dealing with the collision of two nonlinear-elastic half spaces, one of which is addressed, and the second in motion at a constant translational velocity. At the boundary of interaction between the bodies it is assumed that the Coulomb dry-friction law is fulfilled. We examine the nature of the deformation, and we present results from a numerical study of the problem, as well as a comparison with the solution for the linear case.

The Ox_1 axis in the Cartesian coordinate system is directed perpendicular to the half-space boundary (Fig. 1). At the initial instant of time the half space $x_1 > 0$ is fixed,

Voronezh, Kuibyshev. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 30, No. 6, pp. 146-150, November-December, 1989. Original article submitted July 28, 1987; revision submitted August 3, 1988.

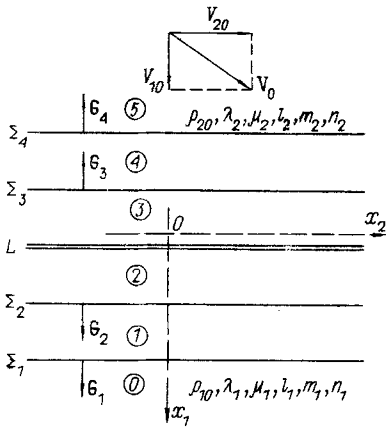

Fig. 1

while when $x_1 < 0$ it has the velocity $V_0 = (V_{10}, V_{20}, 0)$. The material of each of the half spaces, prior to collision, is not subjected to deformation. We will write the system of equations describing the dynamic behavior of the nonlinear-elastic medium in the following form [5, 6]:

$$\begin{gathered}\sigma_{ij} = \frac{\rho}{\rho_0}\frac{\partial W}{\partial e_{ik}}(\delta_{kj} - 2e_{kj}), \quad \sigma_{ij,j} = \rho\frac{dV_i}{dt}, \\ V_i = \partial u_i/\partial t + V_j u_{i,j}, \; 2e_{ij} = u_{i,j} + u_{j,i} - u_{k,i}u_{k,j}, \\ \rho/\rho_0 = (1 - 2I_1 + 2I_1^2 - 2I_2 + 4I_1I_2 - (4/3)I_1^3 - (8/3)I_3)^{1/2}, \\ I_1 = e_{ii}, \; I_2 = e_{ij}e_{ji}, \; I_3 = e_{ik}e_{kj}e_{ji},\end{gathered} \tag{1}$$

where σ_{ij}, e_{ij}, V_i, and u_i are the components of the stress tensors, the Almansi finite strain tensors, of the velocity vectors and the displacement vectors, respectively; ρ and ρ_0 are the densities in the instantaneous and free states. We will use the relationship between the elastic potential W and the invariance of the strain tensor:

$$W = (1/2)\lambda_k I_1^2 + \mu_k I_2 + l_k I_1 I_2 + m_k I_1^3 + n_k I_3, \quad k = 1, \;\; 2. \tag{2}$$

Here λ and μ are the Lamé parameters; ℓ, m, and n are the third-order elasticity moduli; k = 1 for the body against which impact occurs, and k = 2 for the impacting body. At the boundary separating the half spaces L the quantities V_1, σ_{11}, and σ_{12} are continuous

$$[V_1] = [\sigma_{11}] = [\sigma_{12}] = 0, \;\; [F] = F^+ - F^-. \tag{3}$$

The superscripts plus and minus denote the quantities calculated in front of the shock wave (SW) and immediately behind it. The V_2 particle velocity component for the medium will be constant:

$$[V_2] = 0 \tag{4}$$

only on satisfaction at L of the condition

$$|\sigma_{12}| \leqslant f|\sigma_{11}| \tag{5}$$

(f is the coefficient of friction). If the solution for the problem with boundary conditions (3) and (4) fails to satisfy inequality (5), then we will replace (4) with the following relationship:

$$|\sigma_{12}| = f|\sigma_{11}|, \tag{6}$$

which represents the Coulomb dry-friction law. In [2] we find a study of the unique features involved in the deformation of a nonlinear-elastic half space, whose boundary, starting from some instant of time, moves at a constant velocity. Applying the results obtained there to the problem under consideration (the boundary conditions lead to the compression of the medium), we arrive at a wave pattern such as that illustrated in Fig. 1. Thus, the strains generated by the collision of elastic half spaces will propagate through the medium in the form of longitudinal Σ_1, Σ_4 and quasilateral [2] Σ_2, Σ_3 SW, and we will denote the velocities of their motion as G_1, G_2, G_3, G_4.

The discontinuity planes in combination with the plane L separating the colliding bodies divide the elastic space into six zones. In this case, the medium is not deformed in zones 0 and 5. The particles of the medium from zone 5 have constant velocity $\mathbf{V}_0$, while those from zone 0 are nonmoving. In zones 1 and 4 there are no shearing strains $u_{2,1}^{(1)} = u_{2,1}^{(4)} = 0$ nor stresses $\sigma_{12}^{(1)} = \sigma_{12}^{(4)} = 0$ (the superscript indicates that the quantities belong to points in the corresponding zone). At the points in each zone $u_{1,1}$ and $u_{2,1}$ (and consequently, V_i, e_{ij}, σ_{if} as well) are constant. In this case, the equations of motion are satisfied identically. The dynamic and kinematic conditions of consistency [7] are satisfied at the SW fronts:

$$[\sigma_{i1}] = \rho^+(V_1^+\nu_k - G)[V_i], \; i = 1,\, 2; \tag{7}$$

$$\left[\frac{\partial F}{\partial t}\right] + \nu_k G\left[\frac{\partial F}{\partial x_1}\right] = 0, \quad \nu_1 = 1, \quad \nu_2 = -1, \tag{8}$$

which enable us to find the parameters of the stress-strain states at the points in zones 1-4. Using relationships (1), (2), and (8), and limiting ourselves only to the quadratic terms for the tensor components $u_{i,j}$, we obtain

$$[\sigma_{11}] = (\lambda_k + 2\mu_k)\tau_1 + 2\alpha_k u_{1,1}\tau_1 - \alpha_k\tau_1^2 + 2\beta_k u_{2,1}\tau_2 - \beta_k\tau_2^2, \tag{9}$$

$$\begin{gathered}[\sigma_{12}] = \mu_k\tau_2 + \gamma_k(u_{1,1}\tau_2 + u_{2,1}\tau_1 - \tau_1\tau_2);\\ [V_1] = \tau_1(V_1 - \nu_k G)/(1 + \tau_1 - u_{1,1}),\\ [V_2] = (V_1 - \nu_k G)\tau_2 + (u_{2,1} - \tau_2)[V_1],\; \tau_i = [u_{i,1}],\\ \alpha_k = 3(l_k + m_k + n_k) - 3.5\lambda_k - 7\mu_k,\; \gamma_k = l_k + 1.5n_k - \lambda_k - 3\mu_k,\\ \beta_k = (\gamma_k - \mu_k)/2.\end{gathered} \tag{10}$$

Having substituted expressions (9) and (10) into Eq. (7) and taking into consideration that at the waves Σ_1, Σ_4, $[\sigma_{21}] = [V_2] = 0$ we have six algebraic equations

$$\begin{gathered}c_{11}^2[1 + (1 - a_1)\tau_{11}] = G_1^2, \quad c_{11}^2 = (\lambda_1 + 2\mu_1)/\rho_{10}, \quad a_1 = \alpha_1/(\lambda_1 + 2\mu_1),\\ c_{11}^2\left[\tau_{12} + 2(1 - a_1)\tau_{11}\tau_{12} - (a_1 - 1)\tau_{12}^2 - b_1\tau_{22}^2\right] =\\ = G_2^2\tau_{12} - 2G_2(G_1 - G_2)\tau_{11}\tau_{12},\\ c_{21}^2[1 - (d_1 - 1)(\tau_{11} + \tau_{12})] = G_2^2 - 2G_2(G_1 - G_2)\tau_{11},\\ c_{21}^2 = \mu_1/\rho_{10}, \quad b_1 = \beta_1/(\lambda_1 + 2\mu_1), \quad d_1 = \gamma_1/\mu_1,\\ c_{12}^2[1 - (a_2 - 1)\tau_{14}] = (V_{10} + G_4)^2, \quad c_{12}^2 = (\lambda_2 + 2\mu_2)/\rho_{20}, \quad a_2 = \alpha_2/(\lambda_2 + 2\mu_2),\\ c_{12}^2\left[\tau_{13} + 2(1 - a_2)\tau_{13}\tau_{14} - (a_2 - 1)\tau_{13}^2 - b_2\tau_{23}^2\right] = (V_{10} + G_3)^2\tau_{13} -\\ - 2(G_4 - G_3)(G_3 + V_{10})\tau_{13}\tau_{14},\\ c_{22}^2[1 - (d_2 - 1)(\tau_{13} + \tau_{14})] = (V_{10} + G_3)^2 - 2(G_3 + V_{10})(G_4 - G_3)\tau_{14},\\ c_{22}^2 = \mu_2/\rho_{20}, \quad b_2 = \beta_2/(\lambda_2 + 2\mu_2), \quad d_2 = \gamma_2/\mu_2\end{gathered} \tag{11}$$

for ten unknowns τ_{11}, τ_{12}, τ_{13}, τ_{14}, τ_{22}, τ_{23}, G_1, G_2, G_3, G_4. Here, the second of the subscripts for τ indicates for which wave the corresponding discontinuity has been calculated. To obtain a closed system of equations we will use boundary conditions (3)-(6). In the notation adopted above, condition (3) is written in the form

$$\begin{gathered}-\mu_2\tau_{23} + \gamma_2(\tau_{13} + \tau_{14})\tau_{23} = -\mu_1\tau_{22} + \gamma_1(\tau_{11} + \tau_{12})\tau_{22},\\ -(\lambda_2 + 2\mu_2)(\tau_{13} + \tau_{14}) + \alpha_2(\tau_{13} + \tau_{14})^2 + \beta_2\tau_{23}^2 =\\ = -(\lambda_1 + 2\mu_1)(\tau_{11} + \tau_{12}) + \alpha_1(\tau_{11} + \tau_{12})^2 + \beta_1\tau_{22}^2,\\ (V_{10} - G_4\tau_{14} - G_3\tau_{13})(1 + \tau_{11} + \tau_{12}) = (G_1\tau_{11} + G_2\tau_{12})(1 + \tau_{14} + \tau_{13}).\end{gathered} \tag{12}$$

The last relationship closing the system of equations is either the condition of rigid coupling (4)

$$-V_{20} + \left(\frac{V_{10} - G_4\tau_{14}}{1 + \tau_{14}} + G_3\right)\frac{1 + \tau_{14}}{1 + \tau_{13} + \tau_{14}}\tau_{23} = \left(\frac{G_1\tau_{11}}{1 + \tau_{11}} - G_2\right)\frac{1 + \tau_{11}}{1 + \tau_{11} + \tau_{12}}\tau_{22} \tag{13}$$

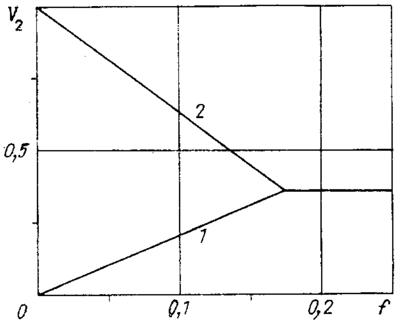

Fig. 2

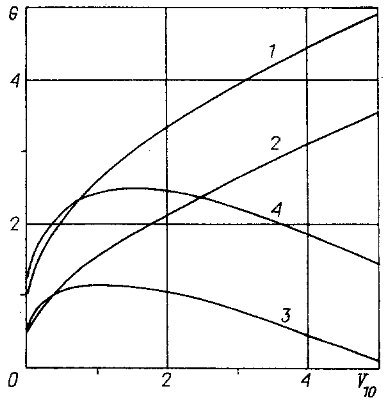

Fig. 3

if the solution will satisfy inequality (5)

$$|-\mu_2\tau_{23}+\gamma_2(\tau_{13}+\tau_{14})\tau_{23}|\leqslant f|-(\lambda_2+2\mu_2)(\tau_{13}+\tau_{14})+\alpha_2(\tau_{13}+\tau_{24})^2+\beta_2\tau_{23}^2|,$$

or if this is not the case, the Coulomb dry-friction law (6)

$$-\mu_2\tau_{23}+\gamma_2(\tau_{13}+\tau_{14})\tau_{23}=f\operatorname{sign}V_2^{(3)}\left[-(\lambda_2+2\mu_2)(\tau_{13}+\tau_{14})+\alpha_2(\tau_{13}+\tau_{14})^2+\beta_2\tau_{23}^2\right]. \quad (14)$$

The derived system of ten nonlinear algebraic equations (11), (12), and (13) or (14) was studied numerically by the Newton iteration method. With small values of V_{10}, V_{20} for the zeroth approximation we took the solution of the problem from linear elasticity theory, which is easily found by analytical methods. For convergence of the iterations at large collision velocities it is necessary gradually to increase V_{10}, V_{20}, taking as the zeroth approximation the solution found in the previous step.

Some of the numerical results are presented in Figs. 2-4 for the case of an aluminum body impacting on a body of steel. In this case the velocities were referred to c_{12} and the stresses to $(\lambda_2 + 2\mu_2)$. The elasticity moduli have the following values: $\lambda_1 = 1.155\cdot 10^5$ MPa, $\mu_1 = 7.7\cdot 10^4$ MPa, $\lambda_2 = 4.05\cdot 10^4$ MPa, $\mu_2 = 2.7\cdot 10^4$ MPa, in Figs. 2 and 4 $\ell_1 = m_1 = n_1 = -10^5$ MPa, $\ell_2 = m_2 = n_2 = -4\cdot 10^4$ MPa, in Fig. 3 $\ell_1 = m_1 = n_1 = -10^6$ MPa, $\ell_2 = m_2 = n_2 = -4\cdot 10^5$ MPa.

Figure 2 shows the velocity components $V_2^{(2)}$ and $V_2^{(3)}$ of the points in the medium (lines 1 and 2) in zones 2 and 3 as functions of the coefficient of friction at the boundary between the bodies, for one and the same velocity of motion $V_{10} = 3$, $V_{20} = 1$ for the impacting body. The connecting point for the graphs of $f = f_*$ corresponds to the onset of coupling for the colliding bodies. The parameters of the stress-strain state, calculated for the case in which $f > f_*$, will be the same as when $f = f_*$.

Figure 3 shows the relationship between the velocities of SW propagation G_1, G_2, G_3, G_4 (lines 1-4) and V_{10} for $f = 0.3$, $V_{20} = 1$. Unlike the linear case in which the SW velocities are constant, these relationships are significantly nonlinear. A reduction in G_3 and G_4 for large V_{10} is explained by the fact that the transfer velocity of the impacting body is directed downward, while the velocities G_3 and G_4 are assumed to be directed vertically upward.

The curves shown in Fig. 4 give the values for the velocities V_{10}, V_{20}, at which plastic deformations set in: 1) $f = 0.1$; 2) $f = 0.3$. These initially arise in aluminum in the region adjacent to the boundary of contact between the colliding bodies. The horizontal segments of the curves correspond to penetration. We made use of the Mises plasticity condition. The results obtained from the application of the Trask plasticity condition differ insignificantly. Plastic strains are developed at collision velocities that are small for problems of the class under consideration (V_{10} = 60-110 m/sec). The calculations were carried out with yield points that have been doubled in comparison to the static values (for steel σ_y = 1500 MPa, and for aluminum σ_y = 600 MPa). Let us note that in our case the process of loading is active, and the appearance of plastic strains cannot therefore be the cause for curtailing the calculations. In this case relationships (1) and (2) may be regarded as a variant of the deformation theory of the plasticity.

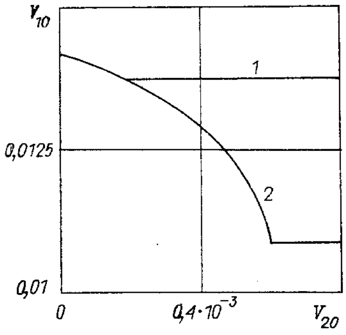

Fig. 4

The question of determining the values of the collision velocities is of interest to the extent that they permit utilization of linear simulation. Numerical experiments have demonstrated that when V_{10} < 100-120 m/sec the relative error between the solutions obtained for the linear-elastic medium and the solution found by means of the nonlinear model does not exceed 5%. Thus, if no plastic strains arise in the collision of elastic bodies, the calculation of the stress-strain state can be accomplished by resorting to the procedures of linear elasticity theory.

The nonlinear effects in the given problem also make themselves evident by virtue of the fact that at the SW in the linear case of purely transverse Σ_2 and Σ_3, the normal components of velocity and stress also undergo discontinuity. In addition, these SW are expansion waves, i.e., we observe the Weissenberg effect (pure shear leads to expansion of the medium). The magnitude of all nonlinear effects increases as V_{10} increases.

Everywhere above, in the description of the results, we make references to intervals in the values of V_{10}. This is explained by the fact that the relationship between the parameters of the strain process and V_{20} is weak and undergoes strengthening only insignificantly as f increases.

It is recommended in [3] for the use of that class of solutions with a quasilateral SW in these problems to verify the satisfaction of the so-called thermodynamic condition of consistency

$$-\rho^{+}(V_n^{+}-G)[V_j][V_j]+2\sigma_{jn}^{+}[V_j]-2(V_n^{+}-G)[W]\geqslant 0. \quad (15)$$

Calculations with various values of the parameters of the problem have shown that the left-hand side of expression (15) is always positive and increases as the velocity of the collision between the bodies increases.

LITERATURE CITED

1. A. A. Burenin, V. V. Lapygin, and A. D. Chernyshov, "The solution of plane self-similar problems in nonlinear dynamic elasticity theory," in: Nonlinear Deformation Waves, Vol. 2 [in Russian], Tallin (1978).
2. A. A. Burenin and V. V. Lapygin, "The self-similar problem of the impact loading of an elastic half space," PMM, 43, No. 4 (1979).
3. A. A. Burenin and V. V. Lapygin, "The reflection of a plane longitudinal shock wave of constant intensity from the plane rigid boundary of a nonlinear elastic medium," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1985).
4. P. F. Sabodash, N. A. Tikhomirov, and I. K. Naval, "Self-similar motions of a physically nonlinear elastic medium, generated by local release of energy," in: Nonlinear Deformation Waves, Vol. 2 [in Russian], Tallin (1978).
5. A. A. Burenin and A. D. Chernyshov, "Shock waves in an isotropically elastic space," PMM, 42, No. 4 (1978).
6. I. I. Gol'denblat, Nonlinear Problems in the Theory of Elasticity [in Russian], Nauka, Moscow (1969).
7. T. Tomas, Plastic Flow and Destruction in Solids [Russian translation], Mir, Moscow (1964).